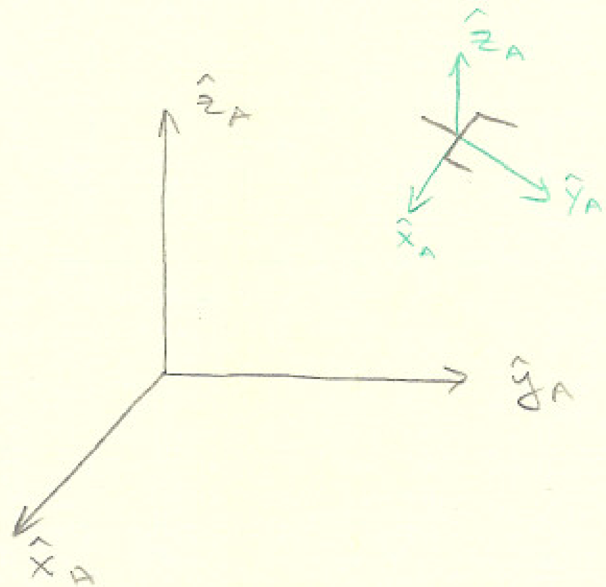
Description of an orientation



The orientation of frame B w.r.t. A is described by three vectors

$$^{A}\hat{X}_B = \begin{bmatrix} r_{11} \\ r_{21} \\ r_{31} \end{bmatrix}$$

$$^{A}\hat{y}_B = \begin{bmatrix} r_{12} \\ r_{22} \\ r_{32} \end{bmatrix}$$

$$^{A}\hat{z}_B = \begin{bmatrix} r_{13} \\ r_{23} \\ r_{33} \end{bmatrix}$$

The components of each vector are a projection of each vector onto the unit direction of the refrence frame A.

We define the rotation matrix of frame B w.r.t. frame A as:

$$ {}^{A}_{B}R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} $$

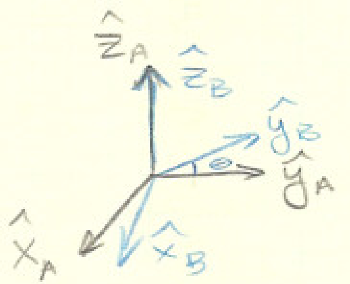
$$ r_{11} = \hat{x}_B \cdot \hat{x}_A $$

$$ r_{21} = \hat{x}_B \cdot \hat{y}_A $$

$$ r_{31} = \hat{x}_B \cdot \hat{z}_A $$

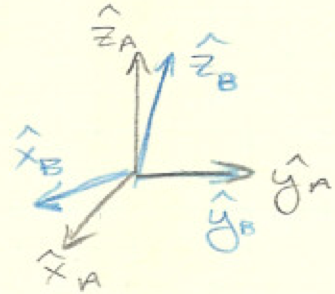
and so on...

EX



$$ \begin{bmatrix} C_\theta & -S_\theta & 0 \\ S_\theta & C_\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} $$

EX



$$ \begin{bmatrix} C_\theta & 0 & S_\theta \\ 0 & 1 & 0 \\ -S_\theta & 0 & C_\theta \end{bmatrix} $$

Note: The inverse of the rotation matrix is the same as the transpose.

$$ {}^{A}_{B}R = \begin{bmatrix} {}^{A}\hat{x}_{B} & {}^{A}\hat{y}_{B} & {}^{A}\hat{z}_{B} \end{bmatrix} $$

$$ = \begin{bmatrix} {}^{B}\hat{x}_{A}{}^{T} \\ {}^{B}\hat{y}_{A}{}^{T} \\ {}^{B}\hat{z}_{A}{}^{T} \end{bmatrix} $$

$$ \therefore \quad {}^{A}_{B}R = {}^{B}_{A}R^{T} = {}^{B}_{A}R^{-1} $$

Note: An orthogonal matrix has its inverse equal to it's transpose.

$$ R_{\hat{z}}(0) = I $$

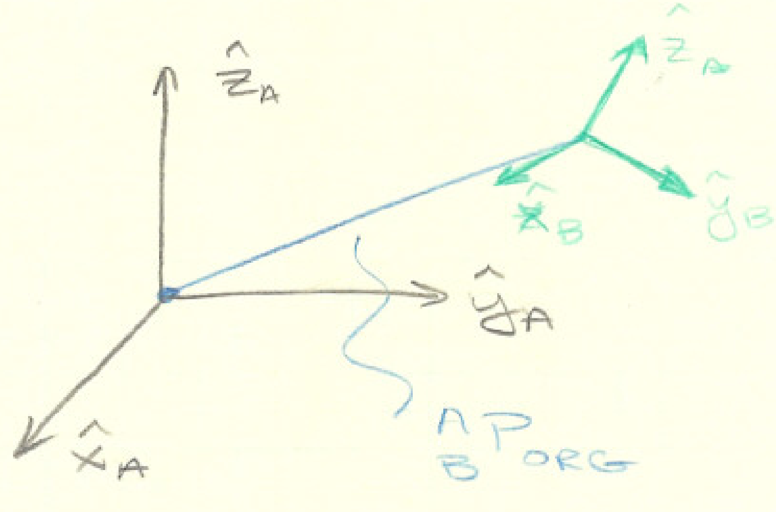
No rotation and we have the identity matrix.

$$ R_{\hat{z}}(\alpha) \cdot R_{\hat{z}}(\theta) = $$

$$ R_{\hat{z}}(\alpha + \theta) $$

$$ R_{z}^{-1}(\theta) = R_{z}(-\theta) $$

Description of a frame

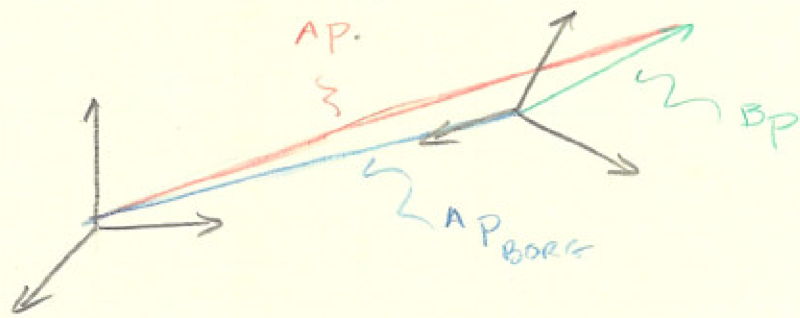


Frame B is described w.r.t. frame A by:

$$ {}^A P_{BORG} $$

$$ {}^A_B R $$

Mapping involving translated feilds



$$ AP = {}^A P_{BORG} + {}^B P $$

Mapping involving rotated feilds

$$ {}^{A}P = {}^{A}_{B}R \, {}^{B}P $$

Mapping involving translations & rotations

$$ {}^{A}P = {}^{A}P_{BORG} + {}^{A}_{B}R \, {}^{B}P $$

We can write the equation here in compact matrix from:

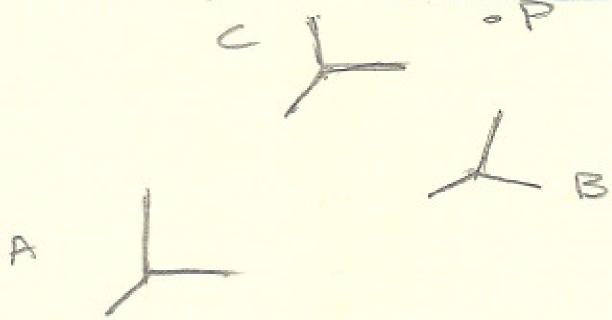
$$ \begin{bmatrix} {}^{A}P \\ 1 \end{bmatrix} = \begin{bmatrix} {}^{A}_{B}R & {}^{A}P_{BORG} \\ 0\ 0\ 0 & 1 \end{bmatrix} \begin{bmatrix} {}^{B}P \\ 1 \end{bmatrix} $$

$$ {}^{A}\overline{P} = {}^{A}_{B}H \, \overline{{}^{B}P} $$

${}^{A}_{B}H$: homogenous matrix.

(AKA ${}^{A}_{B}T$)

Multiple frames Case



$$\overline{BP} = {}^{B}_{C}H \, \overline{CP}$$

$$\overline{AP} = {}^{A}_{B}H \, \overline{BP}$$

$$\therefore \quad \overline{AP} = \underbrace{{}^{A}_{B}H \; {}^{B}_{C}H}_{{}^{A}_{C}H} \; \overline{CP}$$